P425/1
PURE MATHEMATICS
PAPER 1
July/August 2019
3HRS

WESTERN JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

MATHEMATICS

PAPER 1

3 Hours

INSTRUCTIONS TO CANDIDATES.

- *All necessary working must be shown*
- *Section **A** is compulsory*
- *Answer only **five** in section **B***

  ***N.B:** Extra question(s) will not be marked*
- *Un necessary use of calculators/tables will lead to loss of marks.*

SECTION A: (40 MARKS)

Answer *all* questions in this section

1. By using row reduction to echelon form, solve simultaneous equations
   $x + y - z = 1$
   $3x + 4y - 2z = 3$
   $-x$ +y+4z = 2 (05 marks)

2. The line $y = mx$ meets the curve $y^2 = 4x$ at the origin O and at a point A. Find the equation of the locus of the mid-point of OA as m varies. (05 marks)

3. If A, B and C are angles of a triangle, prove that
   $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$ (05 marks)

4. Differentiate$tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to x (05 marks)

5. Find the perpendicular distance of the point $(3, 0, 1)$ from the line whose

   Cartesian equation is$\frac{x-1}{3} = \frac{y+2}{4} = \frac{z}{12}$ (05 marks)

6. Solve the inequality $\left|\frac{1}{1+2x}\right| < 1$ (05 marks)

7. Find $\int_0^{\pi/2} x\, cos^2 3x\ dx$ (05 marks)

8. A cylinder has radius ***r*** and height $8r$. The radius increases from 4cm to 4.1cm; Findthe approximate increase in the volume (use π = 3.14)(05 marks)

SECTION B: (60 MARKS)

Answer any five questions from this section.
All questions carry equal marks

9. (a) IfZ_1 and Z_2 are complex numbers, solve the simultaneous equations $4z_1 + 3z_2 = 23$, $z_1 + iZ_2 = 6 + 8i$giving both answers in the form $x + iy$ (06 marks)

   (b) If $(a + bi)^2 = -5 + 12i$, Find **a** and **b** given that they are both real. Give the two square roots of $-5 + 12i$ (06 marks)

10. (a) Find the equation of the circle which touches the line$3x - 4y = 3$ at the point $(5, 3)$and passes through the point $(-2, 4)$. (05 marks)

    (b) A curve has the parametric equations $x = 3t$, $y = \frac{3}{t}$ Find the equation of the tangent to the curve at the point $\left(3t, \frac{3}{t}\right)$. The point P has coordinates (-5, 8) and the tangents from P to the curve touch the curve at A and B and the length of chord AB (07 marks)

11. (a) If $y = e^{-x}\ In\,x$, show that $x\frac{d^2y}{dx^2} + (2x+1)\frac{dy}{dx} + (x+1)y = 0$ (05 marks)

(b) Express the function $f(x) = \frac{x+2}{(x^2+1)(2x-1)}$ as a sum of partial fractions.

Hence find $\int_2^3 f(x)\,dx$, correct to 4 decimal places (07 marks)

12. Two lines have vector equations

$\vec{r}_1 = \begin{pmatrix} 3 \\ -1 \\ \end{pmatrix} + \lambda 2 \begin{pmatrix} 1 \\ \\ \end{pmatrix}$ and

$\vec{r}_4 + \mu 1 \begin{pmatrix} 4 \\ 1\ 2 \end{pmatrix} \quad -1 \begin{pmatrix} \\ \\ \end{pmatrix}$ Find the position vector

of the point of intersection of the two lines and the cartesian equation of the plane containing the two lines.

(b) Find the acute angle between the line $\frac{x-6}{5} = \frac{y-1}{-1} = \frac{z+1}{1}$ and the plane $7x - y + 5z = -5$, giving your answer to the nearest degree. (05 marks)

13. Find the coordinates of the points ofintersection of the curves.

$y = \frac{x}{x+3}$ and $y = \frac{x}{x^2+1}$

Sketch the cures on the same diagram, showing any asymptotes or turning points.

Show that the area of the finite region in the first quadrant enclosed by the two curves is $\frac{7}{2}\ Ln\,5 - 3Ln\,3 - 2$ (12 marks)

14. (a) In the expansion of $(1+ax)^n$, the first three terms are $1 - \frac{5x}{2} + \frac{75x^2}{8}$

Find $\boldsymbol{n}$ and $\boldsymbol{a}$ and state the range of values of x for which the expansion is valid (06 marks)

(b) Expand $(1+x)^{½}$in ascending powers of x as far as the term in x^2

and hence find an approximation for $\sqrt{1.08}$. Deduce that $\sqrt{12} \approx 3.464$

(06 marks)

15. (a) Solve the equation for $-180^0 \le \theta \le 180^0$,$3 + 2\sin 2\theta = 2sin\theta + 3\ \cos^2\theta$(06 marks)

(b) Giventhat $3\sin x - \cos x = R\sin(x-\alpha)$ *where* $R > 0$ *and* $0^0 < \alpha < 90^0$, Find the values of R and α correct to 1 decimal place.

Hence find one value of x between 0^0 *and* $360^0$for which the curve $y = 3\sin x - \cos x$ has a turning point (06 marks)

16. (a) Find y in terms of x, given that $x\frac{dy}{dx} = cos^2 y$, $x > 0$ and that $y = \frac{\pi}{3}$ *when* $x = 1$(06 marks)

(b) The rate at which a body loses temperature at any instant is proportional to theamount by which the temperature of the body at that instant exceeds the temperature of its surroundings. A container of hot liquid is placed in a room of temperature.18^0C and in 6 minutes the liquid cools from 82^0C to 50^0C.

How long does it take for the liquid to cool from 26^0C to 20^0C? (06 marks)

END

